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Heat transfer coefficients for the laminar fully developed flow of viscoplastic liquids through annuli

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Heat transfer during the laminar fully developed axial flow of viscoplastic materials through concentric annular spaces was studied experimentally. The thermal boundary conditions were insulated outer tube wall and uniform heat flux at the inner tube wall. The flowing liquids were Carbopol aqueous solutions at different concentrations, whose flow curves were well represented by the Herschel–Bulkley viscosity function. The effect of yield stress and power-law exponent on the Nusselt number is investigated. It is shown that the effect of rheological parameters on the inner-wall Nusselt number is rather small.

1. Introduction

Viscoplastic liquids are frequently found in several industrial processes involving food, bio-chemicals, pharmaceuticals, paper, slurries, paint, cosmetics, and petroleum, among others. The rheological behavior of these materials often governs the flows found in these processes. Since convection heat transfer is directly affected by the flow field, then the Nusselt number is expected to be a function of the rheological behavior of the flowing material.

Heat transfer flows in annuli appear frequently in industrial processes. Some examples are the packaging processes of foods, pharmaceutical products, cosmetics and lubricants, the drilling process of oil wells, and the extrusion of ceramic catalyst supports. For this reason, several reports on investigations related to these flows are found in the literature, both for Newtonian fluids, and, to a lesser extent, to non-Newtonian fluids. However, most articles deal with theoretical analyses or numerical simulations [1], while reports on experimental research are rather scarce.

For complex fluids, forced convection inside tubes has been extensively studied. Nouar et al. [2] considered the influence of temperature on the consistency index of a Herschel–Bulkley fluid for the flow through tubes. Soares et al. [3] obtained the Nusselt number both for uniform wall heat flux and uniform wall temperature boundary conditions, while Vradis et al. [4] analyzed the Brinkmann number effect on the Nusselt number. For pseudoplastic liquids, Joshi and Bergles [5] proposed a behavior-index dependent pseudoplastic correction factor for the Newtonian Nusselt number. Shin [6] studied analytically the heat transfer problem in the flow of non-Newtonian fluids in tubes when the thermal conductivity is shear-rate dependent. Scirocco et al. [7] obtained a Nusselt number correlation based on his experimental investigation with pseudoplastic fluids. Several articles deal with the forced convection flow of non-Newtonian liquids where the fluid is heated by the tube wall [8–11]. It is observed that, in the fully developed region, the Nusselt number converges to the values predicted by the correlation suggested by Bird et al. [12].

More recently, Peixinho et al. [13] investigated experimentally the heat transfer to viscoplastic liquids (Carbopol solutions) during the transitional (laminar to turbulent) flow in tubes. They observed that the critical Reynolds number increases as the yield stress is increased, and proposed an extension to the Lévêque model for the laminar regime which accounts for the viscoplastic liquid behavior and temperature dependence of the consistency index.

For flows of non-Newtonian fluids through annuli, heat transfer articles are remarkably scarce. Soares et al. [14] performed a numerical study of the forced convection flow of Herschel–Bulkley fluids through annuli, for boundary conditions of adiabatic outer wall and both uniform temperature and uniform heat flux at the inner wall. They studied the influence on the Nusselt number of the diameter ratio, rheological parameters, Peclet number, and Reynolds number. Exploring their numerical results that indicated the essentially negligible Nusselt number dependence on the rheological properties, Soares et al. [14] were able to develop an analytical expression for the internal Nusselt number for the flow of viscoplastic liquids through annuli.

Nascimento et al. [15] studied four different boundary conditions, and reported the influence of the fluid rheology and diameter ratio on the Nusselt number. Naimi et al. [16] studied experimen-

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tally the forced convection flow of Herschel–Bulkley fluids, and discussed the influence of the temperature-dependent consistency index on the Nusselt number. They also proposed a correlation for the Nusselt number. Nouar et al. [17] presented both numerical and experimental Nusselt number results for the axial flow of a Herschel–Bulkley fluid through an annulus whose inner tube was adiabatic and could rotate, while the outer tube was fixed and exposed to uniform heat flux. Among other findings, the authors concluded that the convective coefficient changed with the variations of the consistency index of the fluid. Nouar et al. [18] also investigated both numerical and experimentally the influence of rheological parameters on the performance of the mixed-convection heat transfer process during the flow of a power-law fluid through the thermal entrance region of annuli. Manglik and Fang [19,20] analyzed the effect of the eccentricity and the behavior index for different diameter ratios and an insulated outer tube, with a thermally active inner tube (both uniform wall temperature and uniform wall heat flux). The authors identified that the larger is the gap, the weaker is the temperature gradient and, consequently, the lower is the convective coefficient.

In the present paper, we report experimental Nusselt number results for the fully-developed axial flow of viscoplastic liquids (Carbopol aqueous solutions) through annuli. The thermal boundary conditions are insulated outer tube wall and uniform heat flux at the inner tube wall. Experimental results for this physical situation could not be found in the open literature. The main goal of this research was to corroborate experimentally the conclusion of a number of independent numerical studies [20,15,14], that the Nusselt number at the inner tube wall is essentially independent of the fluid rheological behavior.

2. Apparatus and experimental procedure

The apparatus was built to allow the measurement of the Nusselt number at the inner tube as a function of the governing dimensionless parameters, to be defined shortly. It is depicted in Fig. 1.

Its main components were the DC power source (maximum heating power of 4032 W), the helicoidal pump (15,000 l/h maximum flow rate), the annular space test section, two Coriolis flow meters installed in parallel to widen the measurement range, and a fluid reservoir.

Aqueous solutions of Carbopol 646 (BF Goodrich) at different mass concentrations (ranging from 0.060% to 0.12%) were employed as working fluids. The flow rate was controlled by varying the electrical frequency (and hence the rotation) of the pump. Because the fluid flowed in closed loop, heat removal was provided to allow steady state measurements.

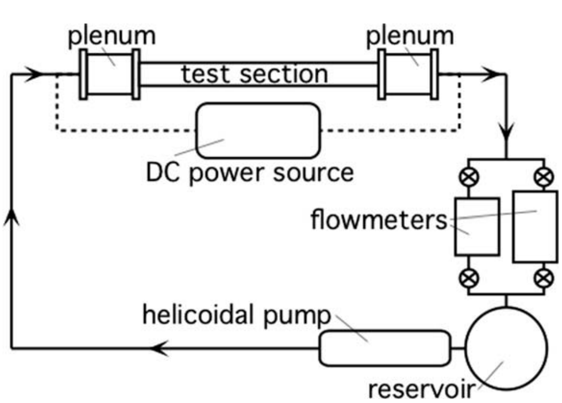

Fig. 1. Schematics of the apparatus.

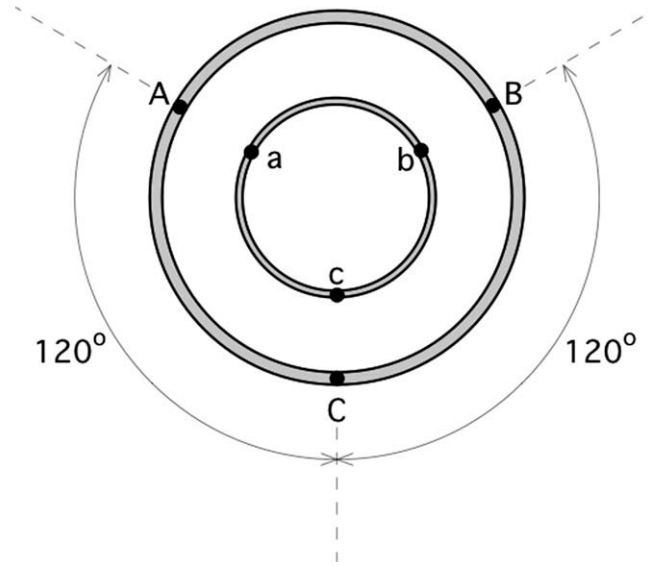

Fig. 2. Thermocouples at external (A–C) and internal (a–c) walls of the annulus.

Calibrated thermocouples (type E) were positioned flush with the surfaces in contact with the fluid (Fig. 2) at three different axial locations 330 mm apart in the fully developed flow region.

Two thermocouples, one positioned at the inlet and the other at the exit of the test section, were used to determine the bulk temperature of the flow at these locations. The annular space was made of two concentric stainless steel tubes. The inner tube was heated by Joule effect, providing the uniform wall heat flux condition, while the outer tube was insulated. The annular test section (Fig. 1) was equipped with two plenums, one at the inlet and the other at the exit. The external diameter of the inner tube was equal to $D_i = 31.75$ mm, while four different external tubes were tested, with nominal diameters of 51, 60, 101, and 220 mm, and wall thicknesses of 2.60 mm, 2.77 mm, 3.05 mm and 2.77 mm, respectively. Thus, three different diameter ratios $\omega \equiv D_i/D_e$ were available: 0.69, 0.58, and 0.33.

2.1. Fluids characterization

The experimental procedure started with the preparation and characterization of the aqueous Carbopol solutions.

It is known that the thermal conductivity of polymeric liquids is influenced by the shear rate when the rate values are larger than 5 s^{-1} [21]. In the present experiments, the shear rate was kept within this limit, and thus the thermal conductivity was assumed to be shear-rate independent. This property was measured in our laboratory with the aid of a concentric-cylinder cell built for this purpose.

A rotational rheometer with a Couette geometry was employed to obtain the flow curve of the Carbopol solutions. In order to reduce wall slip effects, the surfaces in contact with the liquid were covered with sandpaper. The viscosity data were well fitted to the Herschel–Bulkley viscosity function η, given by Eq. (1):

$$\eta(\dot{\gamma}) = \frac{\tau_o}{\dot{\gamma}} + K\dot{\gamma}^{n-1} \quad (1)$$

The parameters that appear in this equation are the yield stress, τ_o; the shear rate, $\dot{\gamma}$; the consistency index, K; and the power-law or behavior index, n. Several flow curves of the Carbopol solutions were obtained for different temperatures in the range of the experiments, and only a mild dependence of the above parameters with temperature was observed. We also checked for fluid degradation by characterizing the fluids just before and just after a data run, but no important degradation effect was observed.

The other thermophysical properties of the solutions were assumed to be equal to that of water at the same temperature, as recommended by Rohsenow and Hartnett [22] for aqueous solutions of less than 1% in mass concentration.

The dimensionless parameters that govern this physical situation are [14] the diameter ratio ω, the power-law index n, and the dimensionless yield stress $\tau_o' \equiv \tau_o/\tau_c$, where τ_c is the characteristic shear stress, defined as:

$$\tau_c \equiv -\frac{dp}{dz}\frac{D_H}{4} = -\frac{dp}{dz}\frac{D_e}{4}(1-\omega) \quad (2)$$

In the above definition, dp/dz is the axial pressure gradient and $D_H = D_e - D_i$ is the hydraulic diameter. The power-law index n is varied by changing the concentration of the Carbopol solution, while τ_o' is a function of both the concentration and the flow rate. In this work, n was varied in the range between 0.4 and 0.8, whereas numerous values of τ_o' were imposed in the range where flow occurs, namely $\tau_o' < 1$ [14].

2.2. Experimental procedure

After charging the system (reservoir, pump, tubes, test section) with the fluid, the pump and power source were turned on and set at the desired flow rate and heating level. Typically, steady state was achieved after about one hour. During each run, all the relevant parameters (temperatures, mass flow rate, heating power, environmental conditions) were registered by the data acquisition system. For each fluid and diameter ratio, runs were performed at different flow rates.

2.3. Data reduction

The heat transfer coefficient and Nusselt number were obtained from the measured quantities as explained next. The heat power Q generated by Joule effect is given by $Q = VI$, where V is the voltage and I the current. Both V and I are measured for each run. The heat flux q is obtained by $q = Q/A$, where A is the heat transfer surface area of the heated tube.

For fully developed flow and the boundary conditions examined, the bulk temperature $T_b(z)$ increases linearly with the axial coordinate z,

$$T_b(z) = T_{b,i} + \frac{q\pi D_i z}{\dot{m}c} \quad (3)$$

where $T_{b,i}$ is the bulk temperature at the inlet of the test section, $\dot{m}$ is the mass flow rate, and c is the specific heat of the flowing liquid.

The local heat transfer coefficient at the thermocouple locations can be determined by

$$h = \frac{q}{T_w(z) - T_b(z)} \quad (4)$$

For fully developed flow and the thermal boundary conditions under consideration, the temperature difference $T_w(z) - T_b(z)$ does not depend on the axial coordinate z, and hence neither does the local heat transfer coefficient h.

The Nusselt number is then evaluated by

$$Nu = \frac{hD_H}{k} \quad (5)$$

where k is the thermal conductivity of the fluid.

The Reynolds number is defined as

$$Re = \frac{\rho\bar{u}D_H}{\eta_c} \quad (6)$$

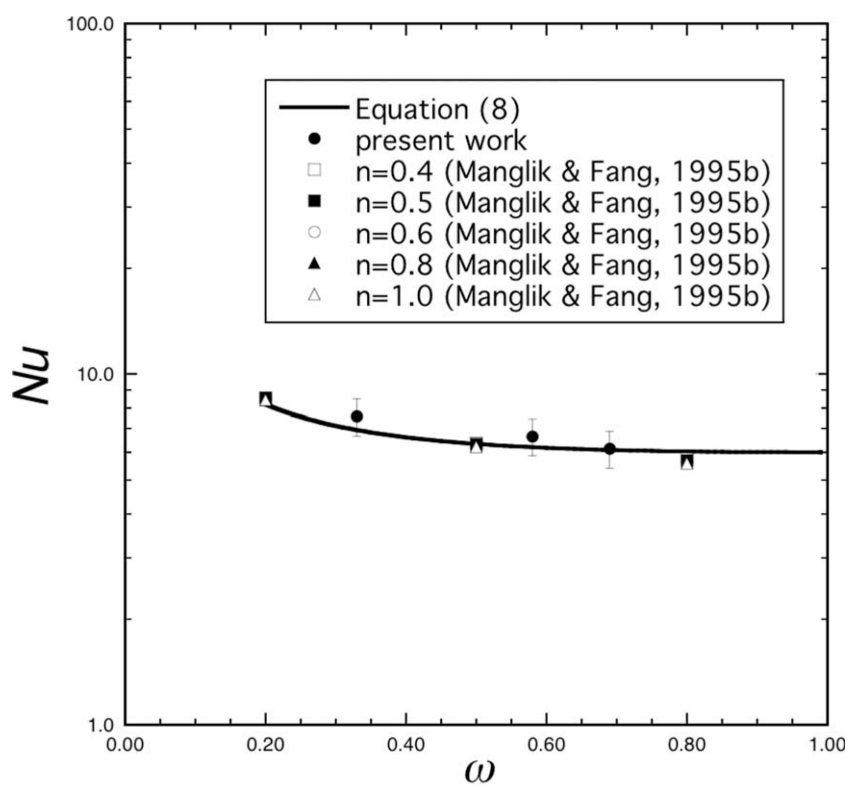

Fig. 3. Nusselt number as a function of the diameter ratio.

where ρ is the mass density, $\bar{u}$ is the average axial velocity, and $\eta_c \equiv \eta(\dot{\gamma}_c)$ is the characteristic viscosity, defined as the viscosity evaluated at the characteristic shear rate $\dot{\gamma}_c$, the latter being defined as:

$$\dot{\gamma}_c \equiv \left(\frac{\tau_R - \tau_o}{K}\right)^{1/n} \tag{7}$$

where τ_R is the inner tube wall shear stress.

To check for possible heating due to mechanical-to-thermal energy conversion, the Brinkmann number $Br \equiv \eta_c \dot{\gamma}_c D_H^2 / k(T_w - T_b)$ was evaluated for all runs, and observed to be always lower than 10^{-3}. The Grashof number $Gr \equiv \rho^2 g \beta (T_w - T_b) D_H^3 / \eta_c^2$ (β is the coefficient of thermal expansion) was also such that $Gr/Re^2 \ll 1$ for all cases, ensuring the absence of natural convection effects in the experiments.

A careful uncertainty analysis for the Nusselt number was programmed in the data reduction spreadsheet to evaluate the *Nu* uncertainty for all cases, and it was observed that its value was typically around 12% or lower.

3. Results and discussion

The experimental results obtained are summarized in Fig. 3. In this figure we plotted the *Nu* data points obtained for three different diameter ratios, namely, $\omega = 0.33$, 0.58, and 0.69. Each of these points is actually an average of values pertaining to different values of *n* and τ_o'. The maximum deviation from these average values was less than 5%, thus much lower than the *Nu* measurement uncertainty $\simeq 12\%$).

The analytical solution for plug flow ($n = 0, \tau_o' = 1$), obtained by Soares et al. [14], is also shown in Fig. 3. This solution is given below:

$$Nu = \frac{8(\omega - 1)(1 - \omega^2)^2}{\omega(\omega^4 - 4\omega^2 + 4\ln\omega + 3)} \tag{8}$$

This *Nu* equation (Eq. (8)) was shown to agree well with results pertaining to all rheological fluid behaviors, ranging from Newtonian to highly viscoplastic or pseudoplastic. Moreover, inner-wall *Nu* results for both uniform wall temperature and uniform wall heat flux are well fitted by Eq. (8) (see Fig. 17 of Ref. [14]). As far as the authors know, the results reported in this work represent the first experimental evidence that the inner-wall Nusselt number is rather insensitive to the fluid rheology.

The numerical results of [20] for pseudoplastic liquids are also plotted in Fig. 3. These results also confirm the insensitivity of *Nu* to the fluid rheology and the consequent applicability of Eq. (8) for any fluid rheology.

4. Conclusions

This paper presented an experimental study of the heat transfer problem for the flow of Herschel–Bulkley materials through concentric annular spaces. The thermal boundary condition for the inner wall was uniform heat flux, while the outer wall was assumed to be adiabatic.

Data were obtained for different Carbopol solutions and flow rates, covering the parameter ranges $0.4 \leqslant n \leqslant 0.8$ and $0 < \tau_o' < 1$. For all cases investigated the inner-wall Nusselt number was observed to be rather insensitive to the rheological behavior of the fluid, as already observed in the numerical investigation of Soares et al. [14]. In this work, a comparison between the extreme cases of Newtonian ($n = 1; \tau_0' = 0$) and plug-flow ($n = 0; \tau_0' = 1$) Nusselt number values show that the inner-wall Nusselt number is given by Eq. (8) for all values of n and τ_o with agreement within 5% for diameter ratios below 0.8. The present results provide the first experimental evidence of these findings.

This fact gives support to the usage of Eq. (8) in practical applications regardless the rheological behavior of the fluid, ranging from Newtonian to highly viscoplastic or pseudoplastic, with acceptable error for most engineering applications.

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